

苏州拟阵理论研讨会

Suzhou Workshop on Matroid Theory



苏州大学数学科学学院

2024年7月7日-12日

Suzhou Workshop on Matroid Theory

Meeting Organizers:

Yi Gu	Soochow University
Yongqiang Liu	USTC
Laurentiu Maxim	University of Wisconsin, Madison
Botong Wang	University of Wisconsin, Madison

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Meeting Venue:

Room 101, Boyuan Building, Tiancizhuang Campus
Soochow University
苏州大学天赐庄校区博远楼 101 室

Meeting Accommodation:

Nanlin Hotel, etc

Suzhou, July 7–12, 2024

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1 Schedule

Time in the schedule is CST (UTC+8:00).

Sunday, July 7

All Day	Registration
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Monday, July 8

9:15–9:30	Opening Ceremony
9:30–10:30	Young-Hoon Kiem Birational geometry of generalized Hessenberg varieties and the Shareshian-Wachs conjecture
Break	
11:00–12:00	Yongqiang Liu L^2 type invariants for hyperplane arrangement complements
Lunch	
14:00–15:00	Laurențu Maxim A geometric perspective on generalized weighted Ehrhart theory
Break	
15:30–16:30	Shiyue Li Multimatroids and rational curves with cyclic action

Tuesday, July 9

9:30–10:30	Christin Bibby Locally geometric posets
Break	
11:00–12:00	June Huh Matroids over Hyperfields, Lorentzian Polynomials, and Related Topics
Lunch	
14:00–15:00	Jian Xiao Polymatroid structure on nef classes
Break	
15:30–16:30	Connor Simpson Polymatroid Schubert varieties

Wednesday, July 10

9:30–10:30	Masahiko Yoshinaga Hyperplane arrangements with non-vanishing homotopy groups
Break	
11:00–12:00	Takuro Abe Double points of line arrangements in the complex projective plane
Lunch	
13:30–17:30	Free Discussion

Thursday, July 11

9:30–10:30	Tom Braden The intersection cohomology module of a matroid
Break	
11:00–12:00	Hailun Zheng The transversal numbers of pure simplicial complexes, polytopes, and spheres
Lunch	
14:00–15:00	Graham Denham Resolving configuration hypersurfaces
Break	
15:30–16:30	Donggun Lee Representations on the cohomology of the moduli space of pointed rational curves, recursion and asymptotic log concavity
18:00–20:00	Banquet

Friday, July 12

9:30–10:30	Colin Crowley The Orlik-Terao algebra and the internal zonotopal algebra of an equivariant hyperplane arrangement
Break	
11:00–12:00	Jacob Matherne Chow theory for posets
Lunch	
14:00–15:00	Arthur Yang Log-concavity of Kazhdan-Lusztig polynomials of uniform matroids
Break	
15:30–16:30	Free Discussion

2 Abstracts

Takuro Abe

Title: Double points of line arrangements in the complex projective plane

Abstract: By the famous Sylvester-Gallai theorem, it is known that every line arrangement in the real projective plane which is not a pencil has a double point. But it is also known that this does not hold true for those in the complex projective plane. However, not so many such arrangements (called the Sylvester-Gallai arrangements) are known. In this talk, we investigate this problem from the viewpoint of algebra of arrangements, mainly the freeness, minimal free distance and the cokernel of the Euler restriction maps.

Christin Bibby

Title: Locally geometric posets

Abstract: When an arrangement of subvarieties locally looks like an arrangement of hyperplanes, its intersection pattern is encoded in a poset which locally looks like a geometric lattice, called a locally geometric poset. We will discuss how various notions related to geometric lattices, including matroids, supersolvability, and building sets, generalize to this setting.

Tom Braden

Title: The intersection cohomology module of a matroid

Abstract: The recent proof of the Dowling-Wilson top-heaviness conjecture for matroids relied on the properties of a module $IH(M)$ associated to a matroid M , particularly the fact that it satisfies the hard Lefschetz property. When M is a realizable matroid, $IH(M)$ has a topological interpretation as the intersection cohomology of the arrangement Schubert variety associated to M . For general matroids, it was defined as a direct summand of the much larger augmented Chow ring $CH(M)$ using a complicated inductive process which is difficult to work with. I will present a simple module-theoretic characterization of $IH(M)$, which also leads to a direct construction avoiding the augmented Chow ring. (Joint work with June Huh, Jacob Matherne, Nicholas Proudfoot and Botong Wang.)

Colin Crowley

Title: The Orlik-Terao algebra and the internal zonotopal algebra of an equivariant hyperplane arrangement

Abstract: In 2017 Moseley, Proudfoot, and Young conjectured that the reduced Orlik-Terao algebra of the braid matroid was isomorphic as a symmetric group representation to the cohomology of a certain configuration space. This was proved by Pagaria in 2022. We generalize Pagaria's result from the braid arrangement to arbitrary hyperplane arrangements and recover a new proof in the case of the braid arrangement. Along the way, we give formulas for several other invariants of a hyperplane arrangement with a group action, including the class of the matroid Schubert variety in the equivariant K-theory of a product of projective lines, and the Postnikov-Shapiro algebra. Work in progress, joint with Nick Proudfoot.

Graham Denham

Title: Resolving configuration hypersurfaces

Abstract: The hypersurface given by the Kirchhoff polynomial is a singular projective variety with relevance to physics, and a special case of a geometric construction from a matroid realization. I will describe a resolution of singularities for such hypersurfaces using a tropical compactification involving some conormal geometry of matroids. This is based on joint work with Dan Bath, Mathias Schulze, and Uli Walther.

June Huh

Title: Matroids over Hyperfields, Lorentzian Polynomials, and Related Topics

Abstract: This presentation aims to explore the relationship between hyperfields and matroids over them, the process of dequantization, and the theory of Lorentzian polynomials. In ongoing joint work with Matt Baker, Mario Kummer, and Oliver Lorscheid, we extend the connection between Lorentzian polynomials and valuated matroids to include matroids over triangular hyperfields, as introduced by Viro. This approach seeks to provide a deeper understanding of the space of Lorentzian polynomials.

Young-Hoon Kiem

Title: Birational geometry of generalized Hessenberg varieties and the Shareshian-Wachs conjecture

Abstract: Regular semisimple Hessenberg varieties have interesting birational morphisms among them which we can exploit to compute the Tymoczko representations on their cohomology. As a consequence, we obtain elementary proofs of the modular law and the Shareshian-Wach conjecture which relates the Frobenius characteristics with the chromatic quasi-symmetric functions of associated graphs. A similar story can be told for the twin manifolds of Hessenberg varieties where the Frobenius characteristics give us certain LLT polynomials. In this talk, I will discuss these birational geometric aspects of Hessenberg varieties and some applications. Based on a joint work with Donggun Lee.

Donggun Lee

Title: Representations on the cohomology of the moduli space of pointed rational curves, recursion and asymptotic log concavity

Abstract: The moduli space of pointed rational curves has a natural action of the symmetric group, which permutes the marked points, inducing a graded representation of the symmetric group on the cohomology of the moduli space.

In this talk, we provide a combinatorial formula for this representation, where we discover a recursive structure that is simple enough to be implemented in computer programming. This is achieved by employing two types of birational models of the moduli space:

- (1) moduli of delta-stable quasimaps, which are certain triples of nodal curves, line bundles and sections, and
- (2) Hassett's moduli of weighted stable curves.

Restricting to the trivial representation part, we derive a new and effective formula for the Poincaré polynomials of the quotient spaces of the moduli spaces by the action of the symmetric groups. We also show that these Poincaré polynomials satisfy asymptotic log-concavity, by establishing asymptotic formulas for the Betti numbers at given degrees. This is based on joint works with J. Choi and Y.-H. Kiem.

Shiyue Li

Title: Multimatroids and rational curves with cyclic action

Abstract: I will share with you a connection between multimatroids and moduli spaces of rational curves with cyclic action. Multimatroids are generalizations of matroids and delta-matroids that naturally arise from topological graph theory. The main result is a combinatorial formula for certain intersection numbers on the moduli space by relating to the volumes of independence polytopal complexes of multimatroids. Based on past and on-going joint works with Emily Clader, Chiara Damiolini, Chris Eur, Daoji Huang, and Rohini Ramadas.

Yongqiang Liu

Title: L^2 type invariants for hyperplane arrangement complements

Abstract: We give formulas for L^2 type invariants of hyperplane arrangement complements at degree one and study their connections with combinatorics. If time allows, some similar results for smooth complex quasi-projective varieties will be discussed.

Jacob Matherne

Title: Chow theory for posets

Abstract: In this talk, I will associate a “Chow polynomial” to any pair (P, κ) consisting of P a finite, graded, bounded poset and κ a certain element of the incidence algebra $I(P)$ called a P -kernel. For the pair where P is the lattice of flats of a matroid and κ is the characteristic polynomial, we recover the Poincaré polynomial of Feichtner-Yuzvinsky’s Chow ring of a matroid (the so-called Chow polynomial of a matroid).

I will discuss general properties of these Chow polynomials as well as connections to other settings such as polytopes and Coxeter groups. In addition, I will explain some relations between these Chow polynomials and the Kazhdan-Lusztig-Stanley polynomials that can also be constructed from such a pair (P, κ) . This is ongoing joint work with Luis Ferroni and Lorenzo Vecchi.

Laurențiu Maxim

Title: A geometric perspective on generalized weighted Ehrhart theory

Abstract: Classical Ehrhart theory for a lattice polytope encodes the relation between the volume of the polytope and the number of lattice points the polytope contains. In this talk, I will discuss a geometric interpretation, via the (equivariant) Hirzebruch-Riemann-Roch formalism, of a generalized weighted Ehrhart theory depending on a homogeneous function on the polytope and with Laurent polynomial weights attached to each of its faces. In the special case when the weights correspond to Stanley's g -function of the polar polytope, we recover in geometric terms a recent combinatorial formula of Beck-Gunnells-Materov. (Based on archive preprints arXiv:2403.17747 and arXiv:2405.02900, joint work with Jörg Schürmann.)

Connor Simpson

Title: Polymatroid Schubert varieties

Abstract: We introduce a stratified singular variety whose geometry captures the polymatroid associated to an arrangement of linear subspaces. This generalizes the "matroid Schubert varieties" associated to hyperplane arrangements. (Joint with C. Crowley and B. Wang)

Jian Xiao

Title: Polymatroid structure on nef classes

Abstract: We discuss the submodularity of numerical dimensions, which induces a polymatroid structure on nef classes on a projective variety. We also explain how the submodularity of the rank function in a polymatroid can be a posteriori put in this geometric framework. The key ingredient is a kind of intersection-theoretic inequalities via Lorentzian polynomials. (Based on joint work with J. Hu)

Arthur Yang

Title: Log-concavity of Kazhdan-Lusztig polynomials of uniform matroids

Abstract: Elias, Proudfoot and Wakefield conjectured that the Kazhdan-Lusztig polynomial of every matroid is log-concave. This interesting conjecture remains widely open. In this talk I will show how to prove this conjecture for uniform matroids and q -uniform matroids. This is based on my joint works with Alice Gao,

Ethan Li, Matthew Xie, Philip Zhang, and Zhong-Xue Zhang.

Masahiko Yoshinaga

Title: Hyperplane arrangements with non-vanishing homotopy groups

Abstract: This talk is based on the preprint: arXiv:2405.20010. Since the discovery of the $K(\pi, 1)$ property for braid arrangements, the homotopy group has called attention of lots of research in the topology of hyperplane arrangements. One of the reasons for the difficulty of studying homotopy groups is that the Hurewicz map always vanishes no matter how the homotopy group is non-vanishing (Randell). In this talk, we will discuss how to detect non-trivial elements of the homotopy group using the twisted intersection numbers.

Hailun Zheng

Title: The transversal numbers of pure simplicial complexes, polytopes, and spheres

Abstract: A transversal of a uniform hypergraph H is a subset of the vertex set that intersects all edges of H . The transversal number $T(H)$ of H is the minimum cardinality of a transversal of H . Transversal numbers of interesting families of hypergraphs have been studied extensively. For example, a classical result of Alon shows that if H ranges over all k -uniform hypergraphs, then

$$\sup \frac{T(H)}{v(H) + e(H)} = \left(1 + o(1)\right) \frac{\ln k}{k},$$

where $v(H)$ denotes the number of vertices and $e(H)$ the number of edges.

In this talk, I will discuss the transversal numbers of hypergraphs that arise from pure simplicial complexes, simplicial polytopes, and simplicial spheres. In particular, I will discuss some new upper and lower bounds, including a few constructions of complexes with relatively large transversal numbers. Many of these constructions are closely related to cyclic polytopes. Joint work with Isabella Novik.

参会名单

List of Participants

Number	Name	Affiliation
1	ABE Takuro	Rikkyo University
2	BIBBY Christin	Louisiana State University
3	BRADEN Tom	University of Massachusetts Amherst
4	CHEN Hong 陈泓	Rutgers University
5	CHU Junyan 储俊妍	Kyushu University
6	CROWLEY Colin	University of Oregon
7	DING Changxin 丁昶欣	Georgia Institute of Technology
8	DENHAM Graham	Western University
9	HUH June	Princeton University
10	KIEM Young-Hoon	KIAS
11	LEE Donggun	IBS Center for Complex Geometry
12	LI Shiyue 李诗乐	IAS, Princeton
13	MAEHARA Shota	Rikkyo University
14	MATHERNE Jacob	North Carolina State University
15	MAXIM Laurentiu	University of Wisconsin-Madison
16	NAKASHIMA Norihiro	Osaka University
17	OBA Ryoshun	University of Tokyo
18	SHEN Wanchun 沈婉春	Harvard University
19	SIMPSON Connor	University of Wisconsin-Madison
20	SUGAWARA Sakumi	Hokkaido University
21	SUN Yuxuan 孙雨璇	University of Minnesota, Twin Cities
22	TAKUYA Saito	Hokkaido University
23	WANG Botong 王博潼	University of Wisconsin-Madison

24	WANG Jidong (Jayden)王季冬	University of Texas at Austin
25	WANG Yiyu 王翌宇	University of Wisconsin-Madison
26	WANG Ying 王滢	University of Michigan
27	WANG Zixuan 王子璇	Hokkaido University
28	WEN Yaxiong 文亚雄	University of Maryland
29	YAMAGATA So	Fukuoka University
30	YAO Yuan 姚远	MIT
31	YOSHINAGA Masahiko	Hokkaido University
32	ZHENG Hailun 郑海伦	University of Hawaii
33	GAO Lulu 郜璐璐	西北工业大学
34	GUI Tao 桂弢	北京大学
35	HU Fei 胡飞	南京大学
36	HU Jiajun 胡佳俊	清华大学
37	LI Jiyou 李吉有	上海交通大学
38	LI Jinyu 李金玉	中国矿业大学
39	LI Yiming 李一鸣	
40	LIU Xinbei 刘心蓓	南开大学
41	LIU Ye 刘晔	西交利物浦大学
42	LIU Yongqiang 刘永强	中国科学技术大学
43	LU Xing 卢兴	清华大学
44	REN Rufeifei 任汝飞	复旦大学
45	WEN Jiayi 温嘉义	香港科技大学
46	WEN Xueqing 文学清	重庆理工大学
47	XIE Kaitao 谢锴涛	香港大学
48	YANG Arthur 杨立波	南开大学
49	ZHANG Biao 张彪	天津师范大学
50	ZHANG Zhongxue 张忠雪	南开大学
51	ZHANG Xutong 张旭彤	中科院数学与系统科学研究院
52	FU Hongyuan 付鸿源	中国海洋大学

53	ZHAO Chengdong 赵承东	中南大学
54	DONG Xinyu 董鑫宇	北京化工大学
55	ZHANG Hongxia 张洪侠	厦门大学
56	XIAO Jian 肖建	清华大学
57	XIE Xun 谢迅	北京理工大学
58	SHEN Hong	
59	ZHANG Ying 张影	苏州大学
60	JI Lijun 季利均	苏州大学
61	GONG Cheng 龚成	苏州大学
62	WANG Kui 王奎	苏州大学
63	BAI Zhanqiang 白占强	苏州大学
64	WANG Xin 汪馨	苏州大学
65	LI Xiangfei 李祥飞	苏州大学

志愿者

Volunteers

1	YI Shichao 易世超	苏州大学
2	SHAN Jiawen 单佳文	苏州大学
3	WANG Zexin 王泽欣	苏州大学