# Recent Developments in Geometric Analysis

11-13 June, 2021

The purpose of this virtual conference is to promote interaction and communications among mathematicians in geometric analysis and related areas all over the world and to improve the understanding of recent progress.

**ZOOM ID: 990 1130 7439** 

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# **Invited Speakers**

Richard Bamler (UC Berkeley)

Jixiang Fu (Fudan University)

Yong Huang (Hunan University)

Mijia Lai (Shanghai Jiao Tong University)

Jiayu Li (University of Science and Technology of China)

Junbin Li (Sun Yat-sen University)

**Gang Liu** (East China Normal University)

**Weimin Sheng** (Zhejiang University)

Xiaokui Yang (Tsinghua University)

**Hui-Chun Zhang** (Sun Yat-sen University)

**Xi Zhang** (University of Science and Technology of China)

Xiao Zhang (Guangxi University & Chinese Academy of Sciences)

**Fangyang Zheng** (Chongqing Normal University)

Yu Zheng (East China Normal University)

Xiaohua Zhu (Peking University)

# **Organizing Committee**

Xiaolong Li (McMaster University)

Lei Ni (UC SanDiego)

Kui Wang (Soochow University)

**Zhenlei Zhang** (Capital Normal University)

## **Sponsors**

Capital Normal University & Soochow University

# Conference Schedule

# June 11, 2021

(09:00-15:00)

Time	Speaker & Title	Host		
09:00—09:50	Richard Bamler  (UC Berkeley)  Compactness and partial regularity theory of Ricci flows in higher dimensions	Bennett Chow (UC SanDiego)		
10:00—10:50	Weimin Sheng (Zhejiang University) Removable singularity of positive mass theorem with continuous metrics			
11:00—11:50	Gang Liu (East China Normal University) Cohn-Vossen inequality on certain noncompact Kahler manifolds			
Break Time				
13:00—13:50	Xiao Zhang  (Guangxi Univ. & Chinese Academy of Sciences)  Spherically symmetric Einstein-scalar-field equations for wave-like decaying null infinity	<b>Zhenlei Zhang</b> (Capital Normal		
14:00—14:50	Hui-Chun Zhang  (Sun Yat-sen University)  Some results for nonlinear geometric analysis on  Alexandrov spaces	University)		

# **Conference Schedule**

# June 12, 2021

(09:00-15:00)

Time	Speaker & Title	Host			
09:00—09:50	Jiayu Li  (University of Science and Technology of China)  Symplectic critical surfaces and its related  evolution	Lei Ni (UC SanDiego)			
10:00—10:50	Xiaohua Zhu (Peking University) Local moduli space of Kahler-Ricci solitons				
11:00—11:50	Xi Zhang  (University of Science and Technology of China)  Some regularity estimates of complex Monge  -Ampere equation				
Break Time					
13:00—13:50	Yong Huang (Hunan University) The Minkowski problem in gaussian probability space	Xiaolong Li (McMaster University)			
14:00—14:50	Mijia Lai (Shanghai Jiao Tong University) Rigidity of spherical domains under curvature lower bound				

# **Conference Schedule**

# June 13, 2021

(09:00-15:00)

Time	Speaker & Title	Host		
09:00—09:50	<b>Jixiang Fu</b> (Fudan University) Limiting of a class of Hermitian-Yang-Mills metrics	Ying Zhang (Soochow University)		
10:00—10:50	Yu Zheng  (East China Normal University)  The recent progress of $\lambda+1$ positive convexity of Riemannian curvature operator along the Ricci flow			
11:00—11:50	Xiaokui Yang  (Tsinghua University)  Strictly nef vector bundles and characterization of projective spaces			
Break Time				
13:00—13:50	Fangyang Zheng (Chongqing Normal University)  Kahler manifolds with positive orthogonal Ricci curvature	Kui Wang (Soochow University)		
14:00—14:50	Junbin Li  (Sun Yat-sen University)  Some mathematical problems of gravitational collapse in general relativity			

#### Compactness and partial regularity theory of Ricci flows in higher dimensions

#### **Richard Bamler**

(UC Berkeley)

**Abstract.** We present a new compactness theory of Ricci flows. This theory states that any sequence of Ricci flows that is pointed in an appropriate sense, subsequentially converges to a synthetic flow. Under a natural non-collapsing condition, this limiting flow is smooth on the complement of a singular set of parabolic codimension at least 4. We furthermore obtain a stratification of the singular set with optimal dimensional bounds depending on the symmetries of the tangent flows. Our methods also imply the corresponding quantitative stratification result and the expected  $L^p$ -curvature bounds. As an application we obtain a description of the singularity formation at the first singular time and a long-time characterization of immortal flows, which generalizes the thick-thin decomposition in dimension 3. We also obtain a backwards pseudolocality theorem and discuss several other applications.

#### Limiting of a class of Hermitian-Yang-Mills metrics

#### **Jixiang Fu**

(Fudan University)

**Abstract.** I will talk about the limiting behavior of a family of Hermitian-Yang-Mills metrics on a class of rank two slope stable vector bundles over a product of two elliptic curves with Kahler metrics which approach a large Kahler metric limit.

#### The Minkowski problem in gaussian probability space

#### **Yong Huang**

(Hunan University)

**Abstract.** In this talk, we will discuss the Minkowski problem in Gaussian probability space. In the solving process, Aleksandrov's variational method leads to an existence result with lagrange multipliers. To avoid it, we provide existence results of smooth solutions with the degree theory of PDE, then an approximation argument is deployed to obtain the weak solution. This is a joint work with Xi Dongmeng (Shanghai University) and Zhao Yiming (MIT).

#### Rigidity of spherical domains under curvature lower bound

#### Mijia Lai

(Shanghai Jiao Tong University)

**Abstract.** In this talk, I will present two rigidity theorems on manifolds with boundary. One is on the spherical domain bounded by Clifford torus under Ricci curvature lower bound, the other is on the hemisphere under Q curvature lower bound. Part of the work is joint with Wei Wei.

#### Symplectic critical surfaces and its related evolution

#### Jiayu Li

(University of Science and Technology of China)

**Abstract.** In this talk, we introduce new functionals to study the existence of holomorphic curves in Kahler surfaces. We will talk about the properties of the critical surfaces of the functionals, and the gradient flow of the functionals.

#### Some mathematical problems of gravitational collapse in general relativity

#### Junbin Li

(Sun Yat-sen University)

**Abstract.** I will give a brief review on the mathematical theories of gravitational collapse in general relativity, and talk about some problems and recent developments, including my works on the formation of trapped surfaces, black holes and instability of naked singularities.

#### Cohn-Vossen inequality on certain noncompact Kahler manifolds

#### **Gang Liu**

(East China Normal University)

**Abstract.** We generalize the Cohn-Vossen inequality to certain noncompact Kahler manifolds. This is related to a conjecture of Yau.

#### Removable singularity of positive mass theorem with continuous metrics

#### **Weimin Sheng**

(Zhejiang University)

**Abstract.** In this talk, I will introduce my recent work with Wenshuai Jiang and Huaiyu Zhang. We consider asymptotically flat Riemannian manifolds endowed with a continuous metric and the metric is smooth away from a compact subset with certain conditions. If the metric is Lipschitz and the scalar curvature is nonnegative away from a closed subset with (n-1)-dimensional Hausdorff measure zero, we show that the ADM mass of each end is nonnegative. Furthermore, if the ADM mass of some end is zero, then we show that the manifold is isometric to the Euclidean space. The Hausdorff measure condition is optimal, which confirms a conjecture of Lee in 2013. The argument of the nonnegativity is based on a smoothing argument with some new estimates such as a  $L^1$ -scalar curvature approximation estimates. The proof of the rigidity relies on the RCD theory where we show that the manifold has nonnegative Ricci curvature in the RCD sense.

#### Strictly nef vector bundles and characterization of projective spaces

#### **Xiaokui Yang**

(Tsinghua University)

**Abstract.** In this talk, we will present some recent progress on the geometry of strictly nef vector bundles, with focus on the characterization of projective spaces.

#### Some results for nonlinear geometric analysis on Alexandrov spaces

#### **Hui-Chun Zhang**

(Sun Yat-sen University)

**Abstract.** Alexandrov spaces are singular spaces with generalized bounds of sectional curvature. In this talk, we will survey some results on geometric analysis on Alexandrov spaces, including two non-linear problems: harmonic maps and variational free boundary problems. This is based on some joint works with Chung-Kwong Chan, and Xi-Ping Zhu.

#### Some regularity estimates of complex Monge-Ampere equation

#### Xi Zhang

(University of Science and Technology of China)

**Abstract.** The complex Monge-Ampere equation has been the subject of intensive studies in the past forty years, because of its important application in complex geometry. In this talk, I will introduce our recent work on regularity estimates of the complex Monge-Ampere equation and their applications. These works are joint with P.F.Guan, C.Li, J.Y.Li, J.W.Liu and X.W.Zhang.

# Spherically symmetric Einstein-scalar-field equations for wave-like decaying null infinity

#### Xiao Zhang

(Guangxi University & Academy of Mathematics and Systems Science, CAS)

**Abstract.** Spherically symmetric Einstein-scalar-field equations have been studied extensively by Christodoulou, which play an important role to understand the global existence and singularity in general relativity. In this talk, we show that the spherically symmetric Einstein-scalar-field equations for wave-like decaying initial data at null infinity have unique global solutions in  $(0,\infty)$  and unique generalized solutions on  $[0,\infty)$  in the sense of Christodoulou. The decaying conditions here are weaker than that in Christodoulou's works and are interesting in gravitational waves. The talk is based on the joint work with Chuxiao Liu.

#### Kahler manifolds with positive orthogonal Ricci curvature

#### **Fangyang Zheng**

(Chongqing Normal University)

**Abstract.** In this talk we report our joint work with Lei Ni and Qingsong Wang on the study of compact Kahler manifolds with positive orthogonal Ricci curvature. We will report on some existence, non-existence, and classification results in low dimensions for this type of manifolds, and also discuss some open questions.

# The recent progress of $\lambda+1$ positive convexity of Riemannian curvature operator along the Ricci flow

#### Yu Zheng

(East China Normal University)

**Abstract.** In this talk, we will first recall our recent year's study on one new  $\lambda+1$  positive convexity of Riemannian curvature operator along the Ricci flow, then some new progress on the differential Harnack inequalities related and problems are introduced. This is a joint work with Xiaodong Cao.

#### Local moduli space of Kahler-Ricci solitons

#### Xiaohua Zhu

(Peking University)

**Abstract.** Let  $(M, J_0)$  be a Fano manifold which admits a Kahler-Ricci soliton  $\omega_{KS}$ . By introducing a suitable parameterized space for Kahler metrics in  $2\pi c_1(M)$ , we first determine the kernel Z for the second variation of Perelman's entropy at  $\omega_{KS}$ . Then we prove that the space Z corresponds to one of Fano manifolds with relatively modified K-semistability near  $(M, J_0)$ . As a consequence, we get a solution of Yau-Tian-Donaldson's conjecture for the existence of Kahler-Ricci solitons in the deformation space of complex structures associated to Z. This is a joint work with Gang Tian and Liang Zhang.